

Gluon Bremsstrahlung in Weakly-Coupled Plasmas

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Abstract

I report on some theoretical progress concerning the calculation of gluon bremsstrahlung for very high energy particles crossing a weakly-coupled quark-gluon plasma. (i) I advertise that two of the several formalisms used to study this problem, the BDMPS-Zakharov formalism and the AMY formalism (the latter used only for infinite, uniform media), can be made equivalent when appropriately formulated. (ii) A standard technique to simplify calculations is to expand in inverse powers of logarithms $\ln(E/T)$. I give an example where such expansions are found to work well for $\omega/T \gtrsim 10$ where ω is the bremsstrahlung gluon energy. (iii) Finally, I report on perturbative calculations of \hat{q} .

1. Equivalence of BDMPS-Zakharov and AMY

The calculation of gluon bremsstrahlung from high-energy particles traversing a quark-gluon plasma is complicated by the Landau-Pomeranchuk Migdal (LPM) effect: at high energy, the formation length for bremsstrahlung (the distance over which gluon or photon emission is coherent) becomes longer than the mean free path between scatterings. As a result, bremsstrahlung from successive scatterings cannot be treated as independent. For QED, Migdal solved this problem in 1956. The basic formalism for treating the LPM effect in QCD was worked out by Baier, Dokshitzer, Mueller, Peigné and Schiff (BDMPS) and Zakharov in 1996-1998 [1]. There have been a number of variations on attacking the problem since then, but the one I want to focus on here is that of myself, Moore, and Yaffe (AMY) [2]. The AMY formalism has been used for the calculation of transport coefficients, such as shear and bulk viscosity, to leading-order in α_s [3, 4]. The AMY formalism was derived independently because, at the time, we did not understand the BDMPS-Zakharov formalism and could not see how to overcome some of its limitations.

One of the limitations of the AMY formalism was that it was formulated only for the case of a uniform, infinite-volume system, which means systems which are approximately uniform over one formation length (and do not change over the corresponding formation time). BDMPS-Zakharov, in contrast, could handle the non-uniform, time-dependent case as well, such as a system whose size is smaller than the formation length. On the downside, BDMPS-Zakharov implicitly treated plasma particles as static scatterers. Furthermore, their results were normalized in terms of the gluon mean free path λ . That's a problem for small coupling expansions because λ is *zero* in (hard-thermal-loop resummed) perturbation theory! A popular model for λ , often used as an example by BDMPS, is perturbative Coulomb scattering cut off by Debye screening, with the form

$$\lambda^{-1} = \int d^2 q_{\perp} \frac{\# g^4 n}{(q_{\perp}^2 + m_D^2)^2}, \quad (1)$$

where q_\perp is the transverse momentum transfer, $n \sim T^3$ is the density of plasma particles, and $m_D \sim gT$ is the Debye mass. The Debye effect screens electric fields, but it does not screen nearly-static components of magnetic fields. As a result, magnetic scattering can take place at much smaller q_\perp , and the actual formula representing perturbative scattering with screening effects is

$$\lambda^{-1} = \int d^2 q_\perp \frac{\# g^2 m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}. \quad (2)$$

This is infrared log divergent, giving $\lambda = 0$.

The result I have to report [5] is that, with minor tweaking of the way BDMPS-Zakharov write their formulas, one can eliminate all the above issues and then easily show that BDMPS-Zakharov reproduces AMY in the uniform, infinite medium limit. So, there is now a BDMPS-Zakharov formalism that handles non-static scatterers, and equivalently there is now a version of AMY for non-uniform media. See Ref. [5] (section II.B and the appendix) for details. The basic tweak to BDMPS-Zakharov is to write their formulas more generally in terms of the rate Γ_{el} for elastic scattering off of the medium instead of as density n times the corresponding cross-section σ , and to avoid normalizing quantities by λ .

2. Large logarithm approximation

To fully evaluate the bremsstrahlung rate to leading order in α_s requires somewhat complicated numerics, even in the case of uniform media. There is a great deal of simplification if one makes the additional approximation that the logarithm $\ln(E/T)$ of energy is large [while still treating $\alpha_s \ln(E/T) \ll 1$]. With this large logarithm approximation, it is possible to get analytic results. To make practical use of a large logarithm expansion, however, one needs a next-to-leading log (NLL) result. Leading log cannot tell the difference, for example, between

$$\ln\left(\frac{E}{T}\right) \quad \text{and} \quad \ln\left(\frac{E}{4\pi^2 T}\right) = \ln\left(\frac{E}{T}\right) + O(1). \quad (3)$$

But these two logarithms are very different for realistic values of E/T .

But, even if one has a NLL result, is it any use in practical situations, or is $\log(E/T)$ never large enough? A test is shown in Fig. 1, which gives results from Ref. [6]. The solid lines are results from a full leading-order evaluation of the $g \rightarrow gg$ bremsstrahlung rate (equivalent to the sort of calculations first performed by Jeon and Moore [7] based on the AMY formalism), as a function of the bremsstrahlung gluon energy $\omega = xE$ divided by T . The dashed lines show the result of the NLL calculation of Ref. [6]. The horizontal axes goes up to the very large, absurdly unrealistic value $\omega/T = 10^5$ just to verify that the curves do approach each other as $\ln(E/T) \rightarrow \infty$ (for fixed x), as they should. The conclusion to take away from this plot is that, in the context of a weakly-coupled plasma, the large logarithm expansion to NLL is good to $\lesssim 20\%$ for $\omega \gtrsim 10T$, which is much better than one might have feared. For $\omega \sim T$, the NLL results is off from the full small-coupling result by roughly a factor of two.

3. \hat{q} in weakly-coupled plasmas

Finally, I want to wrap up this potpourri by discussing the value of the jet broadening parameter \hat{q} in the limit of weak coupling. \hat{q} is defined as the averaged squared transverse momentum

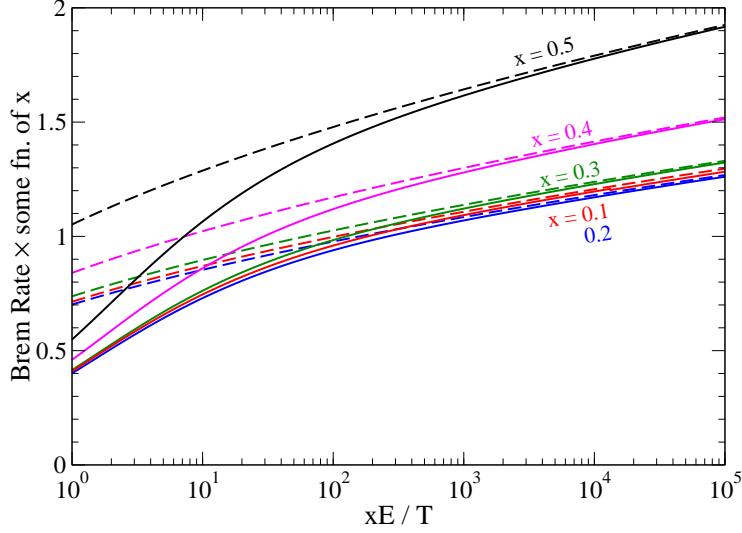


Figure 1: $g \rightarrow gg$ bremsstrahlung rate in an infinite, uniform medium. Solid line is a full calculation to leading order in α_s . Making the further approximation that $\ln(xE/T)$ is large, and working to NLL order, gives the dashed line. See Ref. [6] for normalization of vertical axis.

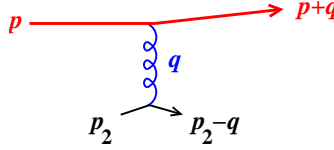


Figure 2: Elastic scattering of a high-energy particle (p) off of a plasma particle (p_2).

transfer Q_\perp^2 per unit length to a high-energy particle traversing the plasma, so that $Q_\perp^2 = \hat{q}L$. It's of relevance to bremsstrahlung because the formation time depends on the collinearity of the bremsstrahlung gluon with the particle that emits it, and the degree of collinearity in turn depends on how much the particles are randomly deflected during the bremsstrahlung process.

The squared transverse momentum transfer per unit length is simply

$$\hat{q} = \int d^2 q_\perp \frac{d\Gamma_{\text{el}}}{d^2 q_\perp} q_\perp^2, \quad (4)$$

where Γ_{el} is the rate for elastic scattering from plasma particles, as in Fig. 2, and q_\perp is the transverse momentum transfer in a single collision. When evaluated at leading-order in α_s (and ignoring the running of α_s), this formula is UV log divergent. For the bremsstrahlung problem, however, what one actually needs is the result $\hat{q}(\Lambda)$ obtained by introducing a UV cut-off Λ on the q_\perp integration [8, 9, 10].

The calculation of the $d\Gamma_{\text{el}}/d^2 q_\perp$ needed in (4), based on the leading-order process of Fig. 2, has the form

$$\frac{d\Gamma_{\text{el}}}{d^2 q_\perp} \sim \int dq_z \int d^3 p_2 \frac{d\sigma_{\text{el}}}{d^3 q} f(\mathbf{p}_2) [1 \pm f(\mathbf{p}_2 - \mathbf{q})], \quad (5)$$

where $f(\mathbf{p})$ is the Bose or Fermi distribution for finding a particle with momentum \mathbf{p} in the

plasma. The $1 \pm f$ factor is a Bose enhancement or Pauli blocking factor for the final state of the plasma particle. Most perturbative calculations of \hat{q} have made additional simplifications to this formula. Some implicitly ignore the final-state $1 \pm f$ factor, which is ignorable only when the dominant momentum transfers q are large compared to T , and even then only to leading-log order. Others replace $1 \pm f(\mathbf{p}_2 - \mathbf{q})$ by $1 \pm f(\mathbf{p}_2)$, valid if the dominant q are small compared to T . To my knowledge, the result has only recently been evaluated using the full form (5) [9, 10]. For $\Lambda \gg T$ [but $\alpha_s \ln(\Lambda/T) \ll 1$], an analytic small-coupling result for UV-regulated \hat{q} is given in Ref. [9]. As an example, for a purely gluonic plasma, it is¹

$$\hat{q}_g(\Lambda) = \left[\zeta(3) \ln \frac{\Lambda}{cT} + \zeta(2) \ln \frac{cT}{m_D} - \sigma_+ \right] \frac{9g^4 T^3}{\pi^3}, \quad (6a)$$

where

$$c \equiv 2 \exp\left(\frac{1}{2} - \gamma_E\right), \quad (6b)$$

$$\sigma_+ \equiv \sum_{k=1}^{\infty} \frac{\ln[(k-1)!]}{k^3} = 0.386043817389949..., \quad (6c)$$

and $m_D = gT$ is the Debye mass. The constant σ_+ can be related to certain generalizations of the Riemann ζ function. I think the expression (6) for \hat{q} is fun and interesting. But if you are of a more practical bent, you could have instead just done the integral (5) numerically.

A presentation of \hat{q} to leading order in α_s requires an important warning: Corrections which are formally higher-order in coupling, of order $m_D/T = O(g)$, have been analyzed by Caron-Huot [10] and are of order 100% for realistic couplings.

Acknowledgments

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¹ For comparison, if one made the $q \ll T$ approximation of replacing $1 + f(\mathbf{p}_2 - \mathbf{q})$ by $1 + f(\mathbf{p}_2)$ in (5), the result would be $\hat{q}_g(\Lambda) = \left[\zeta(2) \ln \frac{\Lambda}{m_D} \right] \frac{9g^4 T^3}{\pi^3}$. If one instead made the $q \gg T$ approximation of dropping the $1+f$ term in (5), the result would be $\hat{q}_g(\Lambda) = \left[\zeta(3) \ln \frac{\Lambda}{m_D} \right] \frac{9g^4 T^3}{\pi^3}$. A rough approximation one sees in the literature is to (i) replace the final factor of q_\perp^2 in (4) by m_D^2 , which artificially eliminates the UV log divergence and gives $\hat{q} \approx m_D^2/\lambda$, and (ii) use the model form (1) for the gluon mean free path λ . This gives $\hat{q}_g \approx \left[\frac{1}{2} \zeta(3) \right] \frac{9g^4 T^3}{\pi^3}$ in the purely gluonic case, for which $\# = 9\zeta(3)/2\pi^4$ in (1).